Registration No.: | | | | | | | | | | |

Total Number of Pages: 02

Course: M.Sc.I
Sub_Code: FMCC701

7th Semester Regular/Back Examination: 2024-25
SUBJECT: Topology
BRANCH(S): M.Sc.I (MC)
Time: 3 Hours
Max Marks: 70
Q.Code: R041
Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)
The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions: **(2 x 10)**

a) Define topology and give an example.
b) Define local connected and path connected in a topological space.
c) Is the set of integers well-ordered? Justify.
d) What is the relation between subspace topology with product topology and subspace topology with ordered topology.
e) Is the set $\left\{x \times \frac{1}{x}: \quad 0 \leq x \leq 1\right\}$ compact? Justify.
f) What is product topology on $X \times Y$.
g) Explain path component with an example.
h) State Tychonoff theorem.
i) Prove that subspace of a first countable space is first countable.
j) State Uryshon lemma.

Part-II

Long Answer Type Questions (Answer Any five)

Q2 a) Show that the subspace (a, b) of $\mathbb{R}$ is homeomorphic with $(0,1)$ and the subspace $[a, b]$ of $\mathbb{R}$ is homeomorphic with $[0,1]$. **(5+5)**
b) Show that the image of connected space under a continuous map is connected.

Q3 a) If $X = \{a, b, c\}$ then show that the collection $\{\{a\}, \{b\}, \{a, b\}, X, \emptyset\}$ is a topology on X. **(5+5)**
b) If $Y = [0, 1] \cup (2, 3)$ is subspace of $\mathbb{R}$, then prove that $E = [0, 1]$ and $F = (2, 3)$ are both open and closed.

Q4 a) Show that every closed interval in $\mathbb{R}$ is compact. **(5+5)**
b) Show that set of rational number Q is not connected.

Q5 a) If X is limiting point compact then show that it is sequentially compact. **(5+5)**
b) Prove or disprove that every limit point compact space is compact.

Q6 **a)** Show that every closed interval in $\mathbb{R}$ is uncountable. **(5+5)**

b) Show that $\mathbb{R}$ is locally compact where as $\mathbb{Q}$ is not.

Q7 **a)** Show that the product of two Lindelof spaces need not be a Lindelof space. **(5+5)**

b) Prove that every compact subspace of a Hausdorff space is closed.

Q8 **a)** Show that every regular space with a countable basis is normal. **(5+5)**

b) Show that a subspace of a completely regular space is completely regular.